# Approaches to Ideal Freeform Mirror and Display Shapes for Augmented Reality ${ }^{1}$ 


#### Abstract

Taking into consideration the physiological properties of human vision, we evaluate optical constraints for a near-eye display (NED) with a single concave mirror and a spatial display. We abandon the usual paradigm that displays have to be flat, and use the display as another free-form optical element. We propose two fundamental geometrical approaches: an off-axis design with elliptic display and mirror shapes and an in-axis design with concentric display and mirror spheres, requiring transparent displays emitting to one side only. We further propose a straightforward construction algorithm for mirrors with arbitrarily shaped displays and we calculate a dome mirror for a planar in-axis display, as a proof of concept. We present precision 3d raytracing results of appearance, fit and optical performance of the various designs.


Near-eye displays have some functional requirements different from other optics designs. A very important fact results from the limited crisp perception area of the human eye, in conjunction with the large angular range required for a versatile near-eye display. A crisp image of a scene can only be acquired cognitively in the brain, by relentless eye movements from detail to detail. To illustrate the consequences for a near-eye display, we regard a simple, single-mirror design as shown in Fig. 1. When a user points his eye towards any particular area on the display, only a tiny fraction of the mirror is involved in the image formation for that particular point. This also differs substantially from classical optics designs in that the real exit pupil of the collimated system is in the center of the eyeball, not in front of the eye pupil.

Fig. 1:
Principle of point formation for NED


In a collimated configuration, i.e. with the virtual image at infinity, the mirror section involved has a diameter of 2 mm for the maximum eye resolution of 1 arcmin . This is simply the pupil size corresponding to this resolution, projected onto the mirror. A smaller pupil causes lower resolution due to beam shaping constraints, a larger pupil also tends to lower resolution, because of aberration.

[^0]Any of the small mirror elements effective for different angles can therefore be optimized for the particular incident beam angle on the mirror, and the focus distance to the corresponding point on the display.
The ideal shape for any such limited area on the mirror would be a properly chosen section of a properly chosen paraboloid. Then we should get an ideally crisp focus point. Alas, the curvature of the entire mirror is not simply the sum of small paraboloids, nor is it a paraboloid itself. We have to find a compromise between the local and the general shape.
The general shape of the mirror would have to be optimized to have all focus points on the display plane, while retaining the best possible approximation to the aforementioned paraboloid shape for any single pupil sized region on the mirror. The concept may work a lot better with a display having a convex shape itself, as indicated in [1].

One of the first explorations, concerning the performance of single mirror approaches as in Fig.1, has been carried out In [2]. It included designs with a naked flat display and with a flat display with a thin correction lens. The angular resolution achieved was about 1.5 arcmin for a design including a lens. For the design without a lens, an explicit number was not given but from the contrast curves it appears to be about 3 arcmin. The fields of view (FOV) provided were about $\pm 10^{\circ}$ ( $20^{\circ}$ total).
This is about what can be expected with planar displays. Some modest improvement may be gained if image distortion is taken care of by pre-compensation in the display image.
We should note that in augmented reality applications, virtual objects will often have to be displayed in the periphery of the field of view, if they are not intended to get in the way of direct sight. A large FOV is therefore always desired.

In military display helmets, large stacks of about a dozen lenses are typically used to prepare the display image. Such stacks are usually placed sideways beneath the head, and the size and weight of the assembly render it impossible for most non-professional applications. A recent approach expands the principle with two large elliptic mirrors, yielding an extremely wide field of view, but at the expense of an even bigger assembly [3].

In this paper, we concentrate on near-eye displays for possible everyday use. We exclude lenses because of their weight. Displays and mirrors can be manufactured almost arbitrarily thin and light, only their size and shape has to be considered.
Non planar displays are introduced in order to provide additional design degrees of freedom, like another optical element. Such displays should soon be in reach with organic substrate and LED technology. Bending displays in one dimensions is already possible even with inorganic technology, provided the functional layers are thin enough. Fully organic displays could also allow for a two dimensional shaping after a planar structuring process.

The first design presented is an off-axis type with an elliptic mirror shape. We will see that this also results in an approximately elliptic display shape, and we will present a basic performance evaluation in terms of resolution and astigmatism.
The second approach proposed uses perfectly spherical mirror and display shapes, concentric to the eye. This requires a unidirectional transparent display and is therefore a bit speculative, but such displays are feasible or at least in reach with current technology, and the concept promises a nearly perfect performance.
Finally, an iterative synthesis method for mirror shapes is shown. It can be used on its own, or for variations of the other designs.

At this point, we should have a look at the requirements for display resolution. Fig. 2 shows an average person's eye resolution as it decreases from the center of view. At the very center, persons with good eyesight resolve up to 1 arcmin. This is the target figure for perfect displays. This maximum resolution, however, needs to be delivered only for the very point the eye is currently looking at. From there, it can be allowed to decline according to Fig. 2. The optical design only has to provide that the crisp area follows the eye rotation.
For the outer ranges of very wide fields of view, even lower requirements apply, as the human eye can only rotate up to a certain limit. The rotation limit is approx. $\pm 45^{\circ}$ horizontal
and vertical. Hence, the periphery of a wide angle near-eye display will always remain in the periphery of the visual field of an eye, and therefore cannot be seen sharp in any case. Fig. 3 shows the limitations of eye rotation and the resulting decrease of resolution outside the maximum rotation range. At least outside of this approx. $45^{\circ}$ range, we may therefore allow for a decreasing display resolution or a degrading optical quality, or both.
average eye resolution


Fig. 2: Eye resolution vs. angle from center [1]


Fig. 3: Maximum range of eye rotation in polar coordinates (may slightly vary for different persons), and max. possible resolutions outside of it, according to Fig. 2

## Calculating optimal freeform mirrors and displays

The first basic design we regard will be the off-axis configuration as in Fig. 1. A spherical mirror shows a severe astigmatism when used at an angle, like in this case. A way of avoiding this is a an elliptic mirror. If we take the eye center as one focus point of the mirror ellipsoid, all rays from there would perfectly converge at the other focus point (Fig. 4).


Fig. 4: Left: an elliptical mirror yields perfect point formation for off-center configurations. Right: if the beam is parallel on one side, a focus of length $f_{2}$ results on the other side

This is not yet exactly what we want, however: while the eye center is a location where all display rays approximately converge, any particular ray bundle coming from the eye pupil has to be assumed parallel, not converging (eye accommodation at infinity).
If we regard the pupil sized area of the ellipsoid mirror involved, it will approximately behave like a simple concave mirror or a lens with focus lengths $f_{1}$ and $f_{2}$ in the directions of the ellipsoid foci $F_{1}$ and $F_{2}$. If we move one of the foci (the one on the eye side) to infinity, the other one gets closer, according to the lens formula $f=1 /\left(1 / f_{1}+1 / f_{2}\right)$.

The calculation returns that all focus points on the display side now come to lie on a smaller surface inscribed in the ellipsoid. The surface obtained this way is almost exactly an ellipsoid, around the same focus points as the mirror ellipsoid. With different values for the position of $F_{2}$ and the mirror radius, we have four discrete parameters for further optimization.

Although this appears to be a geometrically elegant result, we cannot expect it to be the perfectly ideal one for the purpose, because the simple focus distance calculation is correct for the respective center rays at any eye rotation only, hence the peripheral rays do not perfectly converge in the focus point, and the aberration appears to be more of a problem with off-axis designs.

The results obtained, however, are already better than examples known from planar displays, proving the viability of the concept of curved displays.
Insofar as the shapes obtained here can be considerably improved, the general synthesis algorithm proposed at the end of this paper could be considered.

## Dynamic focus adjustment

Dynamic focus adjustment - e.g. by shifting the display back and forward (also with respect to the current viewing direction) - should be an essential part of a display glasses design. With the optical layout presented here, we may ask how big the display displacement would have to be, e.g., for simulating a 40 cm distance?
With $f_{0}$ being the mirror focus length for infinity, and $f_{1}$ the object focus length wanted, we may calculate the according display displacement

$$
\mathrm{d}_{\mathrm{d}}=1 /\left(1 / \mathrm{f}_{0}+1 / \mathrm{f}_{1}\right)-\mathrm{f}_{0}
$$

For $f_{0}=20 \mathrm{~mm}, \mathrm{f}_{1}=400 \mathrm{~mm}$ we get $\mathrm{d}_{\mathrm{d}}=0.95 \mathrm{~mm}$. This is easily manageable. The same displacement, by the way, this would also serve to correct for 2.5 diopters short-sightedness of the user. In conclusion, the design allows for a reasonable focus adjustment range for virtual object distance simulation and also for short- or far-sightedness of the user.


#### Abstract

Aberration It is commonly assumed that the spherical aberration of a pupil sized lens or mirror with a focus length equal to that of the eye pupil will cause about the same spherical aberration and should therefore the system resolution should not turn out lower that that of the eye. One might expect this to hold for off-center designs with elliptic mirrors as well. The elliptic design however results in a considerable focus length change (aberration) from one end of the mirror surface element involved in the point formation to the other. We will see this in the results of the optical simulation.


## Optical Simulation

We will now regard the optical quality of the approach. Image linearity we do not consider, because it can be compensated electronically. What has to be evaluated, is the amount of aberration and astigmatism introduced by the mirror elements when being used for a focus length different from the genuine ellipsoid focus.

We evaluate our designs by precision 3d raytracing, using Blender and the Cycles rendering engine. Blender is a full-featured open source 3D modeling, rendering and animation environment. It also allows for an evaluation of the optical performance, at least of mirror designs. While it cannot replace a specialized optical design software in all detail, the resolution and optical appearance of our designs can be explored almost perfectly. CUDA acceleration allows for very fast, almost real-time rendering with current graphics hardware.

Calculated freeform shapes we import to Blender as vertice lines, by a csv reader plugin. The curves imported are then expanded to 3d shapes by rotating (spinning) them and deleting double vertices when done. Exporting to stl format and re-importing optimizes the surface for rendering. Another very important step is to set 'smooth' rendering for the surface faces.
With these techniques, Blender's camera model delivers accurate optical renderings of displayed image detail to resolutions of 1 arcmin or even below. A camera attached to the eye pupil position allows to explore the virtual images in all aspects.

Displays are simulated by luminescent surfaces, with Image or test pattern textures. The nodes surface model of the Cycles engine also allows for unidirectional emission, as well as back side transparency.

A special advantage of a full 3D modeling is the capability to include detailed anatomic face shapes, allowing to evaluate the visual appearance of the designs and to detect any physical conflicts with the mirrors or displays.

## Simulation results - elliptic display



Fig. 5: ellipsoid design of a free-form NED, here cropped to the useful area for maximum vertical FOV. The displays are showing a full area photo.


Fig. 6: resolution evaluation. Top left: center. Top right: $20^{\circ}$ upward. Bottom left: $20^{\circ}$ left. Bottom right: $20^{\circ}$ down. The overlaid rectangles are 1 arcmin x 1 arcmin.

Fig. 5 shows a complete 3d modeling of the elliptic design, including mirror simulation by ray tracing, and an anatomically correct head model. The glasses and display shaped are but roughly cut (real glasses would include some nicely designed border shaping).
The field of view of the elliptic design can be very large in the horizontal, more than $140^{\circ}$ total, with a binocular center range of $80^{\circ}$. The vertical range, although limited to about $45^{\circ}$, by the display edge getting into sight, is still equal to, or better than many existing designs.

Fig. 6 shows images of a crosshair grid test pattern, with grid lines originally less than 1 arcmin thick. As expected, the display is almost free of astigmatism and the center of view is the location of best relative focus for any eye rotation angle. We see that the optical resolution partly falls short of the desired 1 arcmin. The vertical blur effect may be attributed to the focus change along the vertical extension of the effective mirror area which is certainly an effect present in off-axis designs. For the horizontal, we see a degradation from about one $\operatorname{arcmin}$ at the top to 2 arcmin at the bottom. This may indicate some space for further optimization (we should remember that the ellipse calculation was done by a simple focus law application, and the 2d result has simply been rotated to obtain the 3d shape).

The static peripheral resolution turns out to be more problematic. The simulation yields values only half as good as the peripheral resolution of the eye, or even less, especially if the eye is rotated up or down.

In conclusion, both the center and peripheral resolution of the elliptic off-axis design still do not match the human eye resolution. While the results achieved are better than with planar displays and may still be perceived as reasonably crisp in some applications, we will have to think about ways of improvement.

This could be approached by an adaptation of the general mirror construction method we will propose later in this paper. The limited vertical FOV of the elliptic design, as well as aesthetic aspects, could also be approached by this method.
Dynamic focusing according to eye position would add another degree of freedom for the display and mirror shapes. Yet even with all these approaches, the margins for improvement may be limited.

## Concentric displays

We have seen that the off-center optics design is problematic in terms of optical quality, size and aesthetic appeal. Avoiding the slanted beam angles would be of advantage.
Consider a display that is transparent from its rear side and emits to the front side only. What appears strange at a first glance, can be indeed be achieved, by inserting transparent areas into a single sided display. These can either be gaps in an otherwise non-transparent display structure, or areas of fully transparent driver circuitry. Only the pixel areas have to be occlusive, to cancel out emissions to the rear. Fully transparent notebook displays have been demonstrated several years ago. A single-sided emitting variety has been demonstrated by Toshiba in 2013 (dubbed Transmissive Display).


Fig. 7: Left: a spherical mirror yields perfect point formation at its center (which is trivial). Right: if the incident beam is parallel, a focus of length $\mathrm{r} / 2$ results.

Consequently with such a display, we can abandon the off-center approach. We may use eye-concentric spheres for display and mirror (equivalent to ellipsoids with zero focus point distance). A closer look reveals that there is about one ideal size for these Spheres: 4 cm diameter for the display and 8 cm for the mirror. The focus length of the sphere requires this ratio of $2: 1$ (Fig. 7), while the distance between left and right eye limits the mirror size, because mirrors too big would collide in the middle. The display sphere sizes, on the other hand, should be sufficiently bigger than the eyeball, to achieve a comfortable clearance towards the face.

A complete display design of this type would use only properly chosen sections of the spheres, according to physical limitations by the head shape. A 3d modeling (Fig. 8) shows that with an ample clearance towards face and eyes, almost the entire visual field can still be covered. It also shows the device to look better and be more compact than the elliptic one considered above. Smaller shapes for the mirrors and displays are possible, and should still deliver a sufficient FOV in many applications. A mechanical frame construction will have to be approached separately, and also the integration of further components (eye trackers).

An irritating property of this approach could be the brightness of the display: in order to produce an equivalent visual brightness, it needs to be up to 4 times brighter than the environment. This could occlude the eyes of the user and look quite awkward.
In augmented reality applications, however, only a few virtual objects will usually be present, and they should normally not be in the direct view ahead, as this would obstruct the sight on the real world in a hazardous way.
A possible measure reducing this effect would be limiting the $r, g$ and $b$ colors emitted and reflected to narrow bands (e.g., using dichroic filters and mirrors), which would increase the effectiveness and the visual transparency of the mirror and greatly reduce any light transmission to the outside.


Fig. 8: Photorealistic rendering of spherical glasses. The mirrors are half reflective. The displays in the left image show a very thin test grid pattern (with lines $<1$ arcmin wide).

The gap structure of the transparent display would need to be very fine for our application, according to the targeted resolution of 1 arcmin. For the 2 cm display radius conceived, 1 arcmin means a pixel raster of $6 \mu \mathrm{~m}$. Here we should perhaps start from an entirely transparent design, insert larger gaps between the pixels, and then cover the pixels up from the back side (Fig. 9). This could allow for a high transparency (e.g., 75\%) as shown in Fig. 10 shows the maximum incident angle from the eye towards the display, which should be taken into account for the layer design. With typical OLED emitting layers only $100 \ldots 500 \mathrm{~nm}$ thin, this should not be a problem (Fig. 9). Typical OLED pixels would themselves be transparent and emit to both sides, and the shields would be implemented as mirrors


Fig. 9: Illustration of a transparent display matrix on a spherical glass (left, with greatly enlarged pixels). Matrix and layer detail w. max. beam anlge (right).

The regular micro patterns of the display could lead to diffraction effects, mainly with shallow incident angles. If this would constitute a problem in some cases, introducing some irregularities (dithering) to the display patterns should help.


Fig. 10: Assessment of the max. possible incident angles on the spherical display
An effect occuring with this display type is a faint reflection from the user's eye and skin appearing in the mirror. This image appears very blurred and enlarged, with the pupil always following the center of view (Fig. 11). Given the dark color of the pupil, contrast values should not be greatly affected. If this may be disturbing to some users, will have to be explored in practice.


Fig. 11: left: faint user eye reflection and test grid of 1 arcmin wide lines. Right: display view of a real photo image. Both images showing the FOV of a 24 mm equiv. photo lens (appr. $70^{\circ} \times 40^{\circ}$ ).

The optical quality of the spherical concentric optics design proves to be very high, which does not surprise as the only limiting factor here is the spherical aberration of the mirror areas involved. Fig. 12 shows the simulation results. Indeed, we see the 1 arcmin target is met. This holds for any eye rotation.
The peripheral resolution is $2 . . .6$ times better than that of the eye (cf. Fig. 2). Hence, this display type can achieve close to $100 \%$ quality, both for FOV and resolution.


Fig. 12: Resolution of the concentric spherical display at center (left) and with the eye camera turned right $20^{\circ}$ from the pupil position, assessing the eye peripheral resolution (right). Squares of $1 \operatorname{arcmin} \mathrm{x}$ 1 arcmin are overlaid for comparison. The grid lines displayed are originally <1arcmin wide.

## Exit pupils

The Blender model also allows to simulate a shifting of the entire glasses, e.g., when sliding down the user's nose by a few mm, which is a common problem with traditional glasses frames. Results indicate that for a retention of close to optimum crispness, the elliptic assembly will tolerate only about 2 mm , while the concentric spherical assembly can easily abide 5 mm or more (in one direction, which indicates the total exit pupil is rather about 10 mm ). Resolution in this case will only drop from 1 to about 1.5 arcmin, which is hardly perceivable in practical use.

## A synthesizing method for mirrors for arbitrary display shapes



We will now suggest a straightforward synthesis procedure for mirrors for arbitrary display shapes. It can be used to synthesize entire mirrors, but also to calculate alternative shapes for the edges of otherwise constructed mirrors (such as spherical or elliptical ones).

The underlying idea starts from the fact that the effective beam width for point formation is small (pupil size). Fig. 13 shows the pupil size greatly extended for better illustration. The active mirror section could be approximated, at its edges, by two little planar mirrors ( $m_{1}, m_{2}$ ). The optical effect of said section, focusing a beam of light, can completely be modeled by the two mirrors, reflecting the edge rays of the beam.
The curve between these mirrors will always be very well approximated by a circle sector (with center point $f_{1}$ ), provided the section is chosen to be small enough. The angles of $m_{1}$, $m_{2}$ that we choose to start with, are known. The point where they meet on the display (image point $p_{1}$ ), is then easily constructed.
Now we now rotate the eye by one pupil or beam width. The mirror $m_{2}$, now at the other edge of the beam, gives us the location of a next image point on the display, $\mathrm{p}_{2}$, and this, in turn, dictates the angle of a next mirror element, $\mathrm{m}_{3}$, at the other side of the pupil beam. The spatial position of that next element (its distance from the eye) can perfectly be determined using the sphere sector shape we expect between the mirrors (with center at $f_{2}$ ).
Iterating this method, we get a number of elements approximating a curve, which should turn out to be the ideal mirror shape for that display.

Repeating the entire procedure for eye rotation from the center upwards, we complete the vertical center line of the mirror.
Note that the iteration process may use arbitrary small pupil sizes to improve its resolution and precision.

Next, we may use the same procedure for the horizontal, computing the horizontal center line of the mirror, going from center to the left and to the right. Here we could also apply some constraints if we want to optimize the results for certain display properties (liearity), e.g.:

- keeping the y level constant
- keeping the y position on the display constant

From the crosshair structure acquired by this, we could now fill the entire mirror area.
If we try to combine steps in the horizontal and vertical, however, we will most certainly get conflicting values and will have to somehow average between them, hoping to get a suitable result.
We may also use the initial algorithm from the center in any direction, generating a star - or snow flake - like pattern of mirror elements, defining a complete mirror shape in a most simple way. But this has the disadvantage of neglecting the lateral curvature and introducing more astigmatism. We will see this in the following example.

We should note that the principle of the algorithm can also be used to calculate a display shape for a given mirror shape. This would be a method to calculate a more perfect display shape for the elliptic mirror design treated earlier in paper.

## Mirror synthesis example: Dome mirror and flat display

As a proof of concept, we show a two-dimensional implementation of the mirror synthesis method. It can already be used in practical designs, involving concentric and rotation symmetric components, such as dome mirrors in conjunction with transparent planar displays.
The two-dimensional algorithm needs no programming. It is so straightforward it can indeed be properly implemented even as an Excel sheet. In this example, using smaller pupil sizes for the algorithm yields almost exactly the results as with 2 mm , only smoother. With 0.5 mm , we get a very well resolved curve.
The result we import into the Blender 3d modeling software. We create a dome by rotating the curved mirror shape line with Blender's spinning function. The domes generated are combined with a face model and according display shapes for a comprehensive evaluation.


Fig.14: principle of the planar display - dome mirror design.


Fig. 15: Synthesized freeform mirror example with flat transmissive displays.
Fig. 15 shows the assembly, with two circular flat transparent displays of 29 mm diameter and dome mirrors of 50 mm diameter. This design is not bigger nor heavier than a simple conventional pair of plastic eyeglasses. It allows viewing angles of up to $\pm 35^{\circ}$ in all directions, but astigmatism remains decent only up to $\pm 20^{\circ}$.
The progressive curvature necessary in the radial directions, needed to keep focus on the planar display, in this case is not accompanied by a similar curvature in the circular directions (z coordinate, perpendicular to Fig. 14). This causes growing astigmatism towards the edge. We might think that a three-dimensional mirror synthesis could avoid this, but it won't really work: we would need stronger z curvature to reduce astigmatism, but this can only be achieved by more progressive xy curvature, again requiring even stronger z curvature. Which proves to be a never ending iteration, rendering the problem unsolvable.


Fig. 16: Dome display, center resolution (left). Astigmatism at $20^{\circ}$ eye rotation (middle), compensated by focus change (right). Overlaid rectangles are 1 arcmin $\times 1$ arcmin.

Fig. 16 shows the center resolution of the design; it is almost as good as the concentric sphere design. At $20^{\circ}$ lateral eye rotation, the lateral resolution is still at its optimum, proving the mirror synthesis algorithm to work correctly (this is exactly what it was intended to deliver). The astigmatism, however, is severe, and as argued, this cannot be successfully addressed by changing the mirror shape.

What can be done, is adjusting the local focus, for equal $y$ and $z$ blur. There are 3 ways:

1. Leaving it to the user's eye accommodation, requiring accommodations to, e.g., 1.25 m .
2. Bending the display, by about $1 / 4 \mathrm{~mm}$.
3. Using an anyway desirable dynamic focus feature (moving the display forth and back).

Any of these measures cannot avoid a resolution falloff to approx. 6 arcmin at $20^{\circ}$ eye rotation. This may however be tolerable in some applications also profiting from the really small footprint of these display glasses. The very small bending of $1 / 4 \mathrm{~mm}$ in the second solution however indicates that a moderately curved display (somewhere in between flat and spherical) could provide for a good compromise, yielding very satisfactory edge crispness.

## Toroidal mirror and cylindrical display

We may also use the same xy shape obtained, to combine the spherical concentric principle for the horizontal with a straight shape in the vertical, resulting in a cylindrical display and a toroidal mirror (Fig. 17). This allows for a very large horizontal field of view, and moreover, bending a display in only one direction is far easier than producing an entirely convex one. The astigmatism at the top and bottom may be slightly bigger than with the planar display / dome mirror approach, but together with the very wide horizontal range, the overall performance obtained is quite appealing (Fig. 18). According to Fig. 3, the display could also be straightened a bit at the side edges, up to a limit given by resolution requirements.


Fig. 17: toroidal design with cylindrical display (raw shapes, minimally cut)


Fig. 18: Image simulation of the torus design with a real photo. The bright area marks the FOV of a 24 mm equiv. photo lens (appr. $70^{\circ} \times 40^{\circ}$ )

## Discussion

We have seen that high resolution near eye displays with only one mirror can be achieved if the display itself is used as a functional element in the optical design, with advanced properties such as curved surfaces or back side transparency.
3d raytracing with freely available open source software, proved to be fully able for the simulation of optical resolution as well as physiological and aesthetic design aspects. The approach allows to take care of the physiological workings of human vision, in particular the eye resolution decrease towards the peripheral and the assembly of images by frequent eye movements (rotations). This allows for a much better evaluation than the static eye model often used in classical optics design, and is especially important for near eye displays with a wide field of view.
With the off-center design presented as a first approach, an elliptic mirror shape should be necessary and at least approximately ideal as it minimizes astigmatism. The straightforward calculation of a fitting display shape, using the lens equation, proved to work well in comparison to existing single-mirror-only optics designs, but did not match the high resolution objective of this endeavor. Improvements may be possible with more sophisticated display shape synthesizing methods, like the one proposed later in this paper.

Considerable improvements were obtained by introducing a concept of back side transparent displays, which could be inserted in between the eye and the mirror. Such displays are feasible with current technology and have already been demonstrated as experimental notebook displays.
A design with eye concentric, spherical display and mirror shapes proved to be geometrically and optically ideal, delivering both ultimate crispness and field of view, as well as a large exit pupil, in a well wearable design.
As a two dimensional display curvature is difficult to achieve for the time being, alternatives with flat or simply bent displays were considered, also using the transparent display concept.

A straightforward synthesis method for ideal mirror shapes was developed for this purpose, and applied to mirror designs for flat and cylindrical displays. The flat display variety already delivers good crispness (perfect in the center) for a field of view comparable to existing single-mirror-only optics. The cylindrical display variant adds to this an extremely wide horizontal range.
The objective of 1 arcmin resolution could be achieved for the entire field of view with the spherical design, for an ample center range with the flat transparent display, and in addition to this for the full horizontal range with the cylindrical transparent display. Such high resolution together with light and wearable design, would enable advanced applications, such as the replacement of classical high resolution screens by virtual ones provided in the near-eye display.
The shape synthesis algorithm presented promises to be useful in the further adaptation and refinement of display and mirror shapes in terms of technical feasibility, usability and aesthetic appeal.

Rolf R. Hainich, Jan 15, 2015

[^1]
[^0]:    ${ }^{1}$ All rights preserved. Patents may be pending on approaches presented in this paper.

[^1]:    [1] Rolf R. Hainich; The end of Hardware, Booksurge, 2006/2009.
    [2] [103] O. Cakmakci, S. Vo, S. Vogl, R. Spindelbalker, A. Ferscha, J. P. Rolland: Optical Free-Form Surfaces in Off-Axis HeadWorn Display Design. Proceedings of The 7th IEEE and ACM International Symposium on Mixed and Augmented reality, ISMAR 2008, Cambridge, Sept.15-18, 2008.
    [3] Jianming Yang, Weiqi Liu, Weizhen Lv, Daliang Zhang, Fei He, Zhonglun Wei, Yusi Kang: Method of achieving a wide field-of-view head-mounted display with small distortion. OPTICS LETTERS 203 / Vol. 38, No. 12 / June 15, 2013

